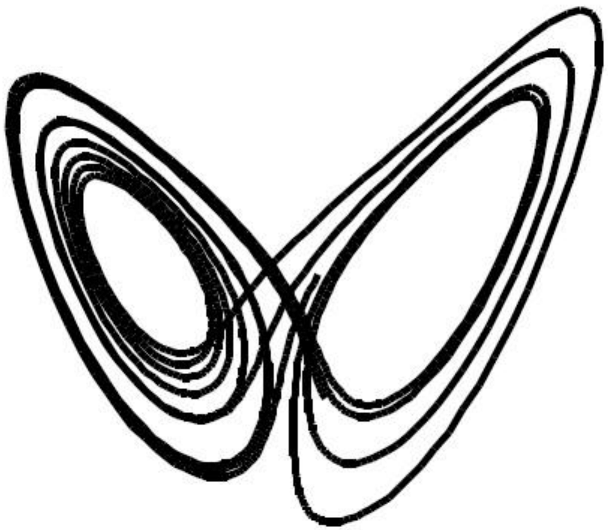

University of Reading

National Centre for Earth Observation
NATURAL ENVIRONMENT RESEARCH COUNCIL

The Kalman Filter

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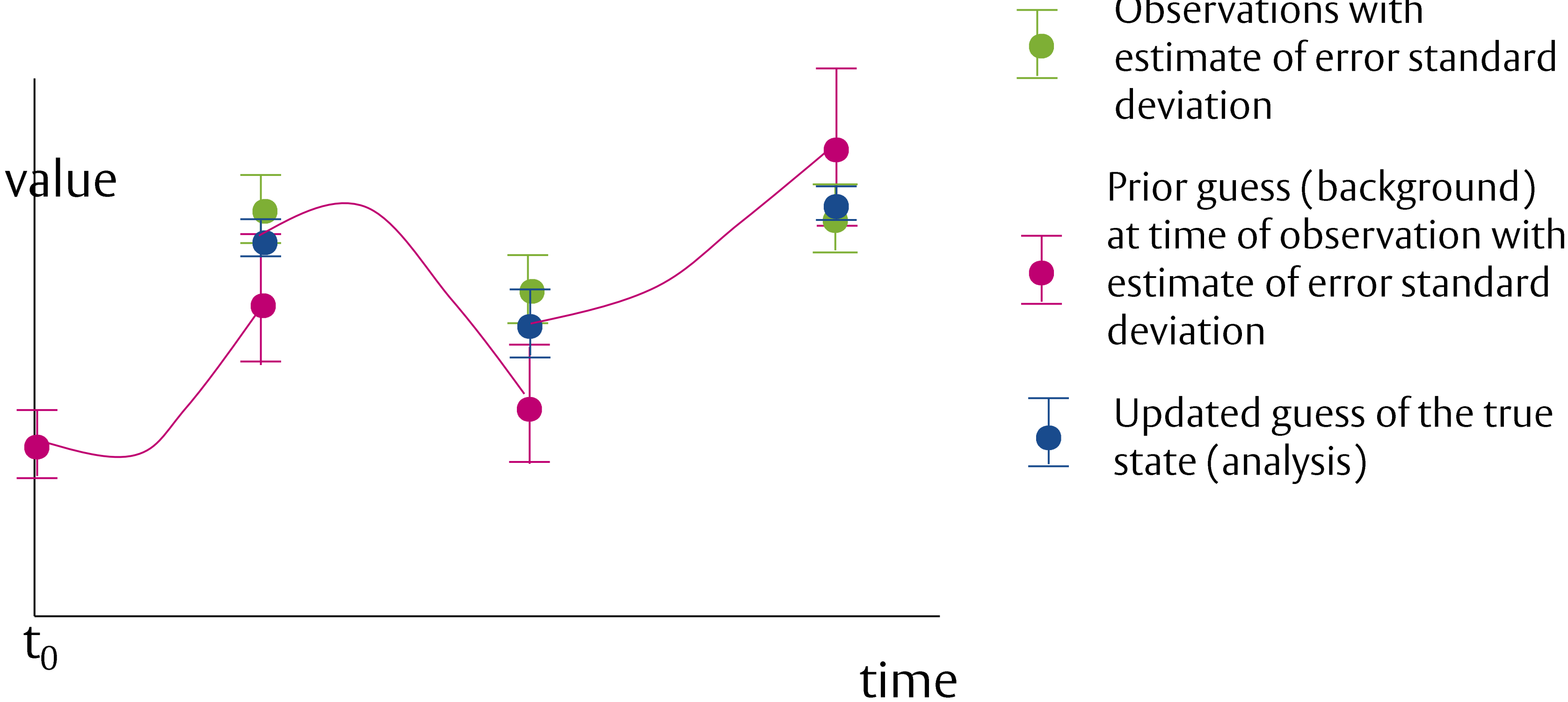
Dynamical data assimilation

- In today and tomorrow's practical we will be looking at methods for assimilating observations made at different times.
- To do this we need to introduce a dynamical model so that we can compare the background, which is valid at an initial time, to the observations.

The Kalman filter

- The Kalman filter assimilates observations sequentially in time.



Schematic of a1D example

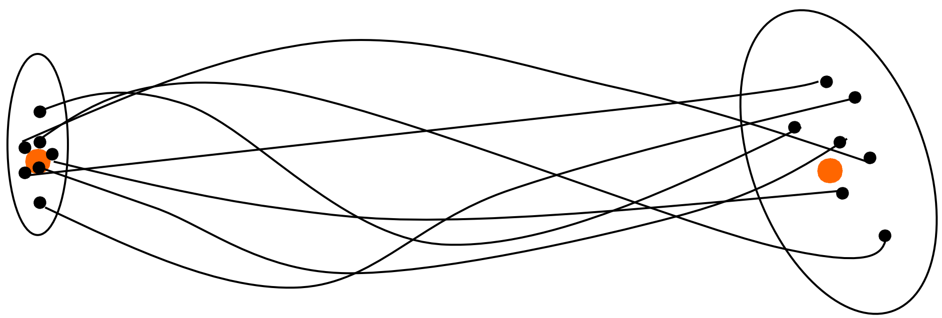
The Kalman Filter

- To assimilate the observations at different times we need to understand how our prior guess of the state (the background) and it's error covariances evolve in time.
- One way is to explicitly evolve them using the dynamical model.
- $\mathbf{x}^{f}(t)=M(\mathbf{x}^{f}(t\text{-}1))$
  - M is our dynamical model
- $\mathbf{P}^{f}(t)=\mathbf{M}\mathbf{P}^{f}(t\text{-}1)\mathbf{M}^{T}+\mathbf{Q}$
  - $\mathbf{M}=\mathrm{d}M(\mathbf{x})/\mathrm{d}\mathbf{x}$ is the tangent linear approximation of M, and $\mathbf{Q}$ is the model error covariance matrix
- This is the Extended Kalman filter method

The Kalman Filter

- Another way of measuring how the error covariances evolve in time is to sample from the initial error distribution and then apply the model to each sample. From this evolved sample we can estimate the error covariances and the mean at the later time.

Initial time: Draw N samples, $\mathbf{x}_i$, from N($\mathbf{x}_b$,$\mathbf{B}$)



At later time:

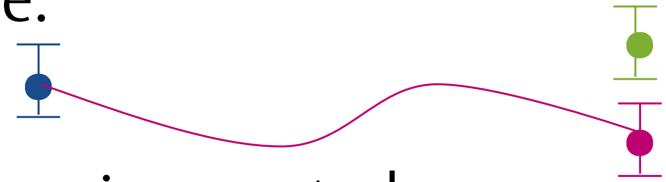
$$\bar{\mathbf{x}} = \frac{1}{N-1}\sum_{i}^{N} M(\mathbf{x}_i)$$

$$\mathbf{P}^{\mathrm{f}} = \frac{1}{N-1}\sum_{i}^{N}(M(\mathbf{x}_i)\text{-}\bar{\mathbf{x}})^{\mathrm{T}}(M(\mathbf{x}_i)\text{-}\bar{\mathbf{x}})$$

- This is the Ensemble Kalman filter method

The update step

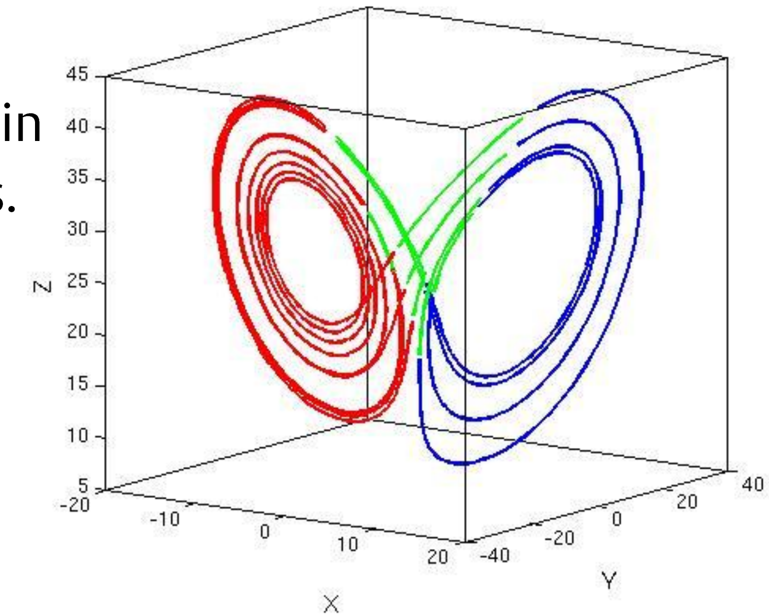
- At the time of the observations we now have the evolved prior mean and error covariance. 
- The calculation of the analysis is similar to the optimal interpolation method introduced in yesterdays practical.  See separate hand out for more details.
- At this stage the analysis error covariance is also calculated. 
- The analysis and analysis error covariances are then evolved to the next observation time to become our prior estimate.



- And the process is repeated

The Dynamical model

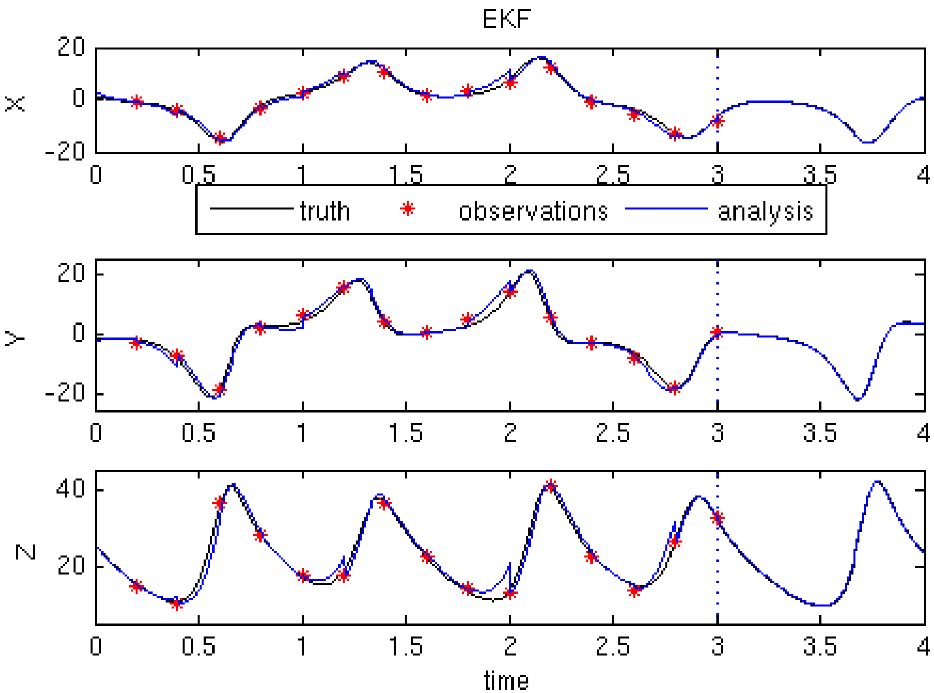
- In this practical we will be using the Lorenz model because
  - It is a toy model with three dimensions allowing us to perform a series of experiments and analyse all aspects of the output
  - It is chaotic, meaning a small error in the initial conditions quickly grows.
  - It orbits around two stable points, giving two ‘regimes’. Accurately forecasting the switch from one regime to another is particularly difficult.



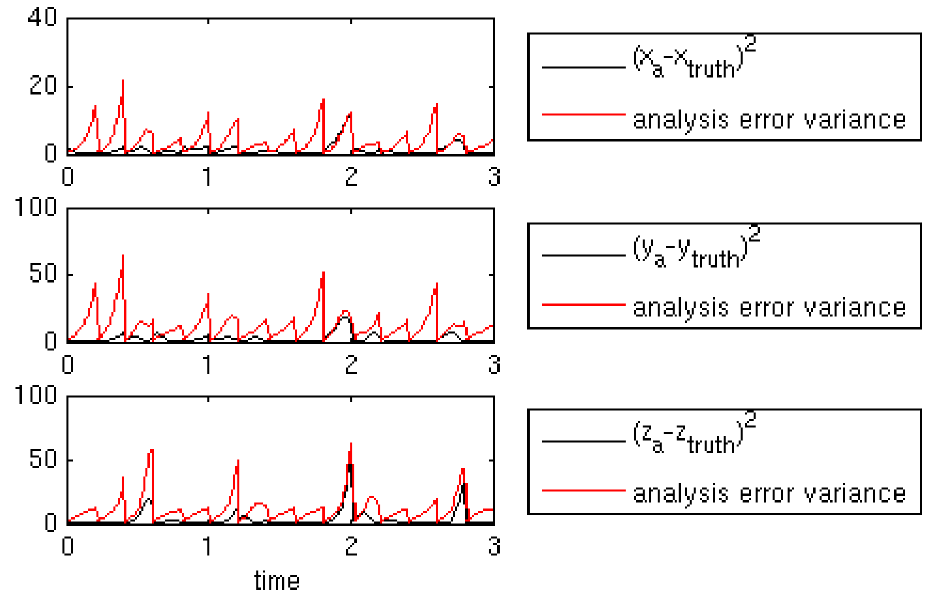
Running the code

- Follow the instructions for running the code.
- You will be given a choice to use the Extended Kalman filter or the Ensemble Kalman filter.
- In each case you must choose the true initial conditions, from this the observations and background will be generated. The analysis trajectory may then be compared to the true trajectory.

Example output- Extended KF



The regime changes can be seen when the x and y values change sign.



Example output- Ensemble KF